ملحق (2)

الصيغ الأساسية للتكامل

1- تكاملات تتعلق بالدالة $a+bx \quad , a\neq 0 \; , b\neq 0$:

$$1)\ \int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx)+c$$

$$2)\ \int (a+bx)^n dx = \frac{(a+bx)^{n+1}}{b(n+1)}+c \ ,\ n\neq -1$$

$$3)\ \int \frac{xdx}{a+bx} = \frac{1}{b^2}[a+bx-a\ln(a+bx)]+c$$

$$4)\ \int \frac{x^2dx}{a+bx} = \frac{1}{b^3}[\frac{1}{2}(a+bx)^2-2a(a+bx)+ + a^2\ln(a+bx)]+c$$

$$5)\ \int \frac{dx}{x(a+bx)} = -\frac{1}{a}\ln\frac{(a+bx)}{x}+c$$

$$6)\ \int \frac{dx}{x^2(a+bx)} = -\frac{1}{ax}+\frac{b}{a^2}\ln\frac{(a+bx)}{x}+c$$

$$7)\ \int \frac{xdx}{(a+bx)^2} = \frac{1}{b^2}[\ln(a+bx)+\frac{a}{a+bx})]+c$$

$$8)\ \int \frac{x^2dx}{(a+bx)^2} = \frac{1}{b^3}[a+bx-2a\ln(a+bx)-\frac{a^2}{a+bx})]$$

$$9)\ \int \frac{xdx}{(a+bx)^3} = \frac{1}{b^2}[\frac{a}{2(a+bx)^2}-\frac{1}{a+bx})]+c$$

2- تكاملات تتعلق بالدوال $a.b \neq 0 \,,\, a^2+x^2 \,,\, a^2-x^2 \,,\, a+bx^2$:

1) $\int \frac{dx}{1+x^2} = Arctgx + c$

2) $\int \frac{dx}{a^2+x^2} = \frac{1}{a} Arctg \frac{x}{a} + c$

3) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \frac{a+x}{a-x} + c$

4) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \frac{x-a}{x+a} + c$

5) $\int \frac{dx}{a+bx^2} = \frac{1}{\sqrt{ab}} Arctg \sqrt{\tfrac{b}{a}}\, x + c \,,\, a > 0 \,,\, b > 0$

6) $\int \frac{dx}{a-bx^2} = \frac{1}{2\sqrt{ab}} \ln \frac{\sqrt{a}+x\sqrt{b}}{\sqrt{a}-x\sqrt{b}} + c$

7) $\int \frac{xdx}{a+bx^2} = \frac{1}{2b} \ln(x^2 + \frac{a}{b}) + c$

8) $\int \frac{x^2dx}{a+bx^2} = \frac{x}{b} - \frac{a}{b} \int \frac{dx}{a+bx^2}$

9) $\int \frac{dx}{x(a+bx^2)} = \frac{1}{2a} \ln \frac{x^2}{a+bx^2} + c$

10) $\int \frac{dx}{x^2(a+bx^2)} = -\frac{1}{ax} - \frac{b}{a} \int \frac{dx}{a+bx^2}$

11) $\int \frac{dx}{(a+bx^2)^2} = \frac{x}{2a(a+bx^2)} + \frac{1}{2a} \int \frac{dx}{a+bx^2}$

12) $$\int \frac{dx}{(x^2-a^2)^m} = -\frac{1}{a^2} \left\{ \frac{x}{(2m-2)(x^2-a^2)^{m-1}} + \frac{2m-3}{2m-2} \int \frac{dx}{(x^2-a^2)^{m-1}} \right\} ,\ m \neq 1$$

3- تكاملات تتعلق بالدالة $\sqrt{a+bx}$, $a.b \neq 0$:

1) $\int \sqrt{a+bx}dx = \frac{2}{3b}\sqrt{(a+bx)^3} + c$

2) $\int x\sqrt{a+bx}\, dx = -\frac{2(2a-3bx)}{15b^2}\sqrt{(a+bx)^3} + c$

3) $\int x^2\sqrt{a+bx}\, dx = \frac{2(8a^2-12abx+15b^2x^2)}{105b^3}\sqrt{(a+bx)^3} + c$

4) $\int \frac{xdx}{\sqrt{a+bx}} = -\frac{2(2a-bx)}{3b^2}\sqrt{a+bx} + c$

5) $\int \frac{x^2dx}{\sqrt{a+bx}}dx = \frac{2(8a^2-4abx+3b^2x^2)}{15b^3}\sqrt{a+bx} + c$

6) $\int \frac{dx}{x\sqrt{a+bx}}dx = \frac{1}{\sqrt{a}}\ln\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}} + c\ ,\ a > 0$

7) $\int \frac{dx}{x\sqrt{a+bx}}dx = \frac{2}{\sqrt{-a}}\, Arctg\sqrt{\frac{a+bx}{-a}} + c\ ,\ a < 0$

8) $\int \frac{dx}{x^2\sqrt{a+bx}}dx = -\frac{\sqrt{a+bx}}{ax} - \frac{b}{2a}\int \frac{dx}{x\sqrt{a+bx}} + c$

9) $\int \frac{\sqrt{a+bx}}{x}dx = 2\sqrt{a+bx} + a\int \frac{dx}{x\sqrt{a+bx}}$

4- تكاملات تتعلق بالدالة $\sqrt{x^2+a^2}$, $a \neq 0$:

1) $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} +$

$+\frac{a^2}{2}\ln(x+\sqrt{x^2+a^2}) + c$

2) $\int\sqrt{(x^2+a^2)^3}\,dx=\frac{x}{8}(2x^2+5a^2)\sqrt{x^2+a^2}+$

$$+\frac{3a^4}{8}\ln(x+\sqrt{x^2+a^2})+c$$

3) $\int x\sqrt{x^2+a^2}\,dx=\frac{1}{3}\sqrt{(x^2+a^2)^3}+c$

4) $\int\frac{dx}{\sqrt{x^2+a^2}}=\ln(x+\sqrt{x^2+a^2})+c$

5) $\int\frac{dx}{\sqrt{(x^2+a^2)^3}}=\frac{x}{a^2\sqrt{x^2+a^2}}+c$

6) $\int\frac{xdx}{\sqrt{x^2+a^2}}=\sqrt{x^2+a^2}+c$

7) $\int\frac{x^2dx}{\sqrt{(x^2+a^2)^3}}=-\frac{x}{\sqrt{x^2+a^2}}+\ln(x+\sqrt{x^2+a^2})+c$

8) $\int\frac{x^2dx}{\sqrt{x^2+a^2}}=-\frac{x}{2}\sqrt{x^2+a^2}-$

$$-\frac{a^2}{2}\ln(x+\sqrt{x^2+a^2})+c$$

9) $\int\frac{dx}{x\sqrt{x^2+a^2}}=\frac{1}{a}+\ln(\frac{x}{x+\sqrt{x^2+a^2}})+c$

10) $\int\frac{dx}{x^2\sqrt{x^2+a^2}}=-\frac{\sqrt{x^2+a^2}}{a^2x}+c$

11) $\int\frac{\sqrt{x^2+a^2}}{x^2}dx=-\frac{\sqrt{x^2+a^2}}{x}+\ln(x+\sqrt{x^2+a^2})+c$

12) $\int \frac{\sqrt{x^2+a^2}}{x}dx = \sqrt{x^2+a^2} - a\ln\frac{a+\sqrt{x^2+a^2}}{x} + c$

5- تكاملات تتعلق بالدالة $\sqrt{a^2-x^2}$, $a \neq 0$:

1) $\int \frac{dx}{\sqrt{1-x^2}} = Arc\sin x + c$

2) $\int \frac{dx}{\sqrt{a^2-x^2}} = Arc\sin\frac{x}{a} + c$

3) $\int \frac{dx}{\sqrt{(a^2-x^2)^3}} = \frac{x}{a^2\sqrt{a^2-x^2}} + c$

4) $\int \frac{xdx}{\sqrt{a^2-x^2}} = -\sqrt{a^2-x^2} + c$

5) $\int \frac{xdx}{\sqrt{(a^2-x^2)^3}} = \frac{1}{\sqrt{a^2-x^2}} + c$

6) $\int \frac{x^2dx}{\sqrt{a^2-x^2}}dx = -\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}Arc\sin\frac{x}{a} + c$

7) $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}Arc\sin\frac{x}{a} + c$

8) $\int x\sqrt{a^2-x^2}dx = -\frac{1}{3}\sqrt{(a^2-x^2)^3} + c$

9) $\int x\sqrt{(a^2-x^2)^3}dx = -\frac{1}{5}\sqrt{(a^2-x^2)^5} + c$

10) $\int x^2\sqrt{a^2-x^2}dx = \frac{x}{8}(2x^2-a^2)\sqrt{a^2-x^2} +$

$+\frac{a^4}{8}Arc\sin\frac{x}{a} + c$

11) $\int \frac{dx}{x\sqrt{a^2-x^2}} = \frac{1}{a}\ln\frac{x}{a+\sqrt{a^2-x^2}} + c$

12) $\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x} + c$

13) $\int \frac{\sqrt{a^2-x^2}\,dx}{x} = \sqrt{a^2-x^2} - a\ln\frac{a+\sqrt{a^2-x^2}}{x} + c$

14) $\int \frac{\sqrt{a^2-x^2}\,dx}{x^2} = -\frac{\sqrt{a^2-x^2}}{x} - Arc\sin\frac{x}{a} + c$

6- تكاملات تتعلق بالدالة $0 \neq a,\ \sqrt{x^2-a^2}$:

1) $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln(x+\sqrt{x^2-a^2}) + c$

2) $\int \frac{dx}{\sqrt{(x^2-a^2)^3}} = -\frac{x}{a^2\sqrt{x^2-a^2}} + c$

3) $\int \frac{xdx}{\sqrt{x^2-a^2}} = \sqrt{x^2-a^2} + c$

4) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{a}\sqrt{x^2-a^2} -$

$-\frac{a^2}{2}\ln(x+\sqrt{x^2-a^2}) + c$

5) $\int x\sqrt{x^2-a^2}\,dx = \frac{1}{3}\sqrt{(x^2-a^2)^3} + c$

6) $\int x\sqrt{(x^2-a^2)^3}\,dx = \frac{1}{5}\sqrt{(x^2-a^2)^5} + c$

7) $\int \frac{x^2dx}{\sqrt{x^2-a^2}} = \frac{x}{2}\sqrt{x^2-a^2} + \frac{a^2}{2}\ln(x+\sqrt{x^2-a^2}) + c$

8) $\int \frac{dx}{x\sqrt{x^2-1}} = Arc\cos x + c$

9) $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} Arc\cos \frac{x}{a} + c$

10) $\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2 x} + c$

11) $\int \frac{\sqrt{x^2-a^2}}{x} dx = \sqrt{x^2-a^2} - aArc\cos \frac{a}{x} + c$

13) $\int \frac{\sqrt{x^2-a^2}}{x^2} dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln(x+\sqrt{x^2-a^2}) + c$

7- تكاملات تتعلق بالدالة $(a+bx+cx^2, \quad c>0, a.b \neq 0)$:

1) $\int \frac{dx}{a+bx+cx^2} = \frac{2}{\sqrt{4ac-b^2}} Arctg \frac{2cx+b}{\sqrt{4ac-b^2}} + c,$

$b^2 < 4ac$

2) $\int \frac{dx}{a+bx+cx^2} = \frac{1}{\sqrt{b^2-4ac}} \ln \frac{2cx+b-\sqrt{b^2-4ac}}{2cx+b+\sqrt{b^2-4ac}} + c,$

$b^2 > 4ac$

3) $\int \sqrt{a+bx+cx^2}\, dx = \frac{2cx+b}{4c}\sqrt{a+bx+cx^2} -$

$- \frac{b^2-4ac}{8\sqrt{c^3}} \ln(2cx+b+2\sqrt{c}\sqrt{a+bx+cx^2}) + c$

4) $\int \frac{xdx}{\sqrt{a+bx+cx^2}} = \frac{1}{c}\sqrt{a+bx+cx^2} -$

$- \frac{b}{2\sqrt{c^3}} \ln(2cx+b+2\sqrt{c}\sqrt{a+bx+cx^2}) + c$

8- تكاملات تتعلق بالدالة $(a+bx-cx^2\ ,\ \ c>0, a.b\neq 0)$:

1) $\int\frac{dx}{a+bx-cx^2}=$

$$=\frac{1}{\sqrt{b^2+4ac}}\ln(\frac{\sqrt{b^2+4ac}+2cx-b}{\sqrt{b^2+4ac}-2cx+b}+c$$

2) $\int\frac{dx}{\sqrt{a+bx-cx^2}}=$

$$=\frac{1}{\sqrt{c}}Arc\sin(\frac{2cx-b}{\sqrt{b^2+4ac}})+c$$

3) $\int\sqrt{a+bx-cx^2}\,dx=\frac{2cx-b}{4c}\sqrt{a+bx-cx^2}$

$$=\frac{b^2+4ac}{8\sqrt{c^3}}Arc\sin(\frac{2cx-b}{\sqrt{b^2+4ac}})+c$$

4) $\int\frac{xdx}{\sqrt{a+bx-cx^2}}=-\frac{\sqrt{a+bx-cx^2}}{c}+$

$$+\frac{b}{2\sqrt{c^3}}Arc\sin(\frac{2cx-b}{\sqrt{b^2+4ac}})+c$$

9- تكاملات تتعلق بدوال مختلفة :

1) $\int\sqrt{\frac{a+x}{b+x}}dx=\sqrt{(a+x)(b+x)}+$

$$+(a-b)\ln(\sqrt{a+x}+\sqrt{b+x})+c$$

2) $\int\sqrt{\frac{a-x}{b+x}}dx=\sqrt{(a-x)(b+x)}+$

$$+(a+b)Arc\sin(\sqrt{\frac{x+b}{a+b}})+c$$

$$3)\int\sqrt{\frac{a+x}{b-x}}dx=-\sqrt{(a+x)(b-x)}+$$

$$-(a+b)Arc\sin(\sqrt{\frac{b-x}{a+b}})+c$$

$$4)\int\sqrt{\frac{1+x}{1-x}}dx=-\sqrt{1-x^2}+Arc\sin x+c$$

$$5)\int\frac{dx}{\sqrt{(x-a)x-b)}}=2Arc\sin\sqrt{\frac{x-a}{b-a}}+c$$

10- تكاملات تتعلق بدوال أسية ومثلثية وقطعية :

$$1)\int a^x dx=\frac{a^x}{\ln a}+c \quad , \quad 2)\int e^{ax}dx=\frac{e^{ax}}{a}+c$$

$$3)\int e^x dx=e^x+c \quad , \quad 4)\int\sin xdx=-\cos x+c$$

$$5)\int\cos xdx=\sin x+c \quad , \quad 6)\int tgxdx=-\ln\cos x+c$$

$$7)\int ctgxdx=\ln\sin x+c$$

$$8)\int sc\,xdx=\ln(sc\,x+tgx)+c=\ln tg(\frac{x}{2}+\frac{\pi}{4})+c$$

$$9)\int\csc xdx=\ln(\csc x-ctgx)+c=\ln tg\frac{x}{2}+c$$

$$10)\int\sin^2 xdx=\frac{x}{2}-\frac{1}{4}\sin 2x+c$$

$$11)\int\cos^2 xdx=\frac{x}{2}+\frac{1}{4}\sin 2x+c$$

$$12)\int\sin^n xdx=-\frac{\sin^{n-1}x\cos x}{n}+\frac{n-1}{n}\int\sin^{n-2}xdx+c$$

13) $\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x dx + c$

14) $\int \frac{dx}{\sin x} dx = \ln\left| tg \frac{x}{2} \right| + c$

15) $\int \frac{dx}{\sin^n x} dx = -\frac{1}{n-1} \times \frac{\cos x}{\sin^{n-1} x}$

$$+\frac{n-2}{n-1}\int \frac{dx}{\sin^{n-2} x} + c \ , \ n \neq 1$$

16) $\int \frac{dx}{\cos x} dx = \ln\left| tg(\frac{x}{2} + \frac{\pi}{4}) \right| + c$

17) $\int \frac{dx}{\cos^n x} dx = \frac{1}{n-1} \times \frac{\sin x}{\cos^{n-1} x}$

$$+\frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} x} + c \ , \ n \neq 1$$

18) $\int \sin x \cos x dx = \frac{\sin^2 x}{2} + c$

19) $\int \sin x \cos^n x dx = -\frac{\sin^{n+1} x}{n+1} + c \ , \ n \neq -1$

20) $\int \sin^n x \cos x dx = \frac{\sin^{n+1} x}{n+1} + c \ , \ n \neq -1$

21) $\int \sin^n x \cos^m x dx = \frac{\cos^{n-1} \sin^{m+1} x}{m+n} +$

$$+\frac{m-1}{m+n}\int \cos^{m-2} x \sin^n x dx + c$$

22) $\int \sin mx \sin nx dx = -\frac{\sin(m+n)x}{2(m+n)}$

$$+\frac{\sin(m-n)x}{2(m-n)} + c \ , \ m \neq n$$

23) $\int \cos mx \cos nx dx = \dfrac{\sin(m+n)x}{2(m+n)}$

$+ \dfrac{\sin(m-n)x}{2(m-n)} + c \ , \quad m \neq n$

24) $\int x e^{ax} dx = \dfrac{e^{ax}}{a^2}(ax-1) + c$

25) $\int x^n e^{ax} dx = \dfrac{x^n e^{ax}}{a} - \dfrac{n}{a} \int x^{n-1} e^{ax} dx$

26) $\int u.dv = u.v - \int v du$

27) $\int \ln x dx = x(\ln x - 1) + c$

28) $\int \dfrac{dx}{x \ln x} = \ln(\ln x) + c$

29) $\int x^n \ln x dx = x^{n+1}\left[\dfrac{\ln x}{n+1} - \dfrac{1}{(n+1)^2}\right] + c$

30) $\int \ln^n x dx = x \ln^n x - n \int \ln^{n-1} x dx$

31) $\int x^m \ln^n x dx = \dfrac{x^{m+1}}{m+1} \ln^n x -$

$- \dfrac{n}{m+1} \int x^m \ln^2 x dx \ , \quad m \neq -1$

32) $\int e^x \sin x dx = \dfrac{e^x(\sin x - \cos x)}{2} + c$

33) $\int e^{ax} \sin nx dx = \dfrac{e^{ax}(a \sin nx - n \cos nx)}{a^2 + n^2} + c$

34) $\int e^x \cos x dx = \dfrac{e^x(\sin x + \cos x)}{2} + c$

35) $\int e^{ax}\cos nx dx = \dfrac{e^{ax}(n\sin nx + a\cos nx)}{a^2+n^2} + c$

36) $\int_0^{\infty} \dfrac{x^{\alpha-1}}{1+x}dx = \dfrac{\pi}{\sin\alpha\pi} \quad , \quad 0<\alpha<1$

37) $\int_0^{\pi} (1+\cos x)^{\alpha-1}(1-\cos x)^{-\alpha}\sin x dx = \dfrac{\pi}{\sin\alpha\pi}$

38) $\int shx.dx = chx + c$

39) $\int chx.dx = shx + c$

- بعض دساتير التحويل المثلثية والقطعية :

$$shx = \frac{e^x - e^{-x}}{2} \quad , \quad chx = \frac{e^x + e^{-x}}{2} \quad , \quad ch^2x - sh^2x = 1$$

$$thx = \frac{shx}{chx} \quad , \quad cthx = \frac{chx}{shx} \quad , \quad sh2x = 2shxchx$$

$$sh(x\pm y) = shxchx \pm chxshx$$

$$ch(x\pm y) = chxchy \pm shxshy$$

$$\sin x\cos y = \tfrac{1}{2}[\sin(x+y) + \sin(x-y)]$$

$$\cos x\sin y = \tfrac{1}{2}[\sin(x+y) - \sin(x-y)]$$

$$\cos x\cos y = \tfrac{1}{2}[\cos(x+y) + \cos(x-y)]$$

$$\sin x\sin y = -\tfrac{1}{2}[\cos(x+y) - \cos(x-y)]$$

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